TRINITY COLLEGE

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Semester Two Examination, 2022

Question/Answer booklet

MATHEMATICS METHODS UNITS 3&4

Section Two: Calculator-assumed

If required by your examination administrator, please place your student identification label in this box

WA student number: In figures

In words

Number of additional answer booklets used (if applicable):

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	52	35
Section Two: Calculator-assumed	12	12	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of Trinity College examinations are detailed in the *Instructions to Candidates* distributed to students prior to the examinations. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed **65% (98 Marks)**

This section has **twelve** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 8 **(7 marks)**

Naltrexone is useful in managing heroin-dependent patients who find it difficult to shift away from dependent use patterns. The blood naltrexone level N of a patient who has received a naltrexone implant was observed to halve every 33 days, from an initial level of 7.7 ng/ml. The level can be modelled by an equation of the form $N = ae^{kt}$, where t is the time in days since the implant was received.

(a) State the value of the constant a and then determine the value of the constant k. (3 marks)

The treatment is effective whilst the naltrexone level remains above 1.6 ng/ml.

(b) Determine the number of days that the implant will be effective. (2 marks)

(c) Determine the rate at which the naltrexone level is decreasing 15 days after the implant is received. (2 marks)

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Question 9 **(8 marks)**

The launch speed of a small projectile fired from a catapult was measured and found to be normally distributed with a mean of 17.4 ms^{-1} and a standard deviation of 0.21 ms^{-1}.

(a) Determine the probability that the projectile is launched with a speed less than 17 ms^{-1}. (1 mark)

(b) Determine the probability that the projectile is launched with a speed less than 17.5 ms^{-1} given that its launch speed exceeds 17 ms^{-1}. (2 marks)

(c) In a series of 15 launches, determine the probability that the speed of the projectile is less than 17 ms^{-1} in at least 2 of these launches. (2 marks)

(d) The projectile is expected to have a speed exceeding v ms once in every 80 launches. Determine the value of v. (1 mark)

(e) The instrument used to measure the launch speed was suspected to underestimate the speed of the projectile by 0.04 ms^{-1}. If this was the case, state the true mean and standard deviation of the distribution of launch speeds for the projectile. (2 marks)

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Question 10 **(6 marks)**

The owners of a shopping mall wanted to confirm their estimate that 35% of local school students visited their mall at least once a week. The owners considered the following three ways of selecting a sample:

A Ask students who turn up to the mall after school.

B Create an online survey and publish a link to it in the local newspaper.

(a) Briefly discuss a source of bias in each sampling method and suggest a better sampling procedure. (3 marks)

(b) It was found that 105 out of a random sample of 375 students visited the mall at least once a week. Determine the 95% confidence interval for the proportion based on this data and use it to comment on the owner's estimate. (3 marks)

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Question 11 **(8 marks)**

(a) A polynomial function is defined by $f(x) = (kx - 1)^3$, where k is a constant. The area under the curve $y = f(x)$ between $x = 3$ and $x = 9$ is 12 square units.

Determine the area under the curve $y = f(x)$ between $x = 3$ and $x = 6$. (4 marks)

(b) The graph of another polynomial $y = g(x)$ has a point of inflection at $(3, 7)$ and a stationary point when $x = -1$.

If $g'(x) = 3x^2 + ax + b$, where a and b are constants, determine $g(x)$. (4 marks)

Question 12 **(10 marks)**

An online retailer of auto parts knows that on average, 18.5% of parts sold will be returned.

(a) Let the random variable X be the number of parts returned when a batch of 88 parts are sold.

(i) Describe the distribution of X. (2 marks)

(ii) Determine the probability that less than 15% of the parts sold in this batch will be returned. (2 marks)

The retailer takes a large number of random samples of 150 parts from its sales data and records the proportion $\hat{p}$ of returned parts in each sample. Under certain circumstances, the distribution of $\hat{p}$ will approximate normality.

(b) Explain why the retailer can expect the distribution of $\hat{p}$ to closely approximate normality in this case. (3 marks)

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(c) State the parameters of the normal distribution that $\hat{p}$ approximates and use this distribution to determine the probability that the proportion of returns in a random sample of 150 parts is less than 15%. (3 marks)

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Question 13 **(8 marks)**

Brass ingots are cast by a metal recycling machine with masses of X kg, where X is a continuous random variable with cumulative distribution function

$$F(x) = \begin{cases} 0 & x < 3 \\ ax^2 - bx & 3 \leq x \leq 4 \\ 1 & x > 4 \end{cases}$$

(a) Deduce from the cumulative distribution function that the values of the constants a and b are $a = 0.25$ and $b = 0.75$. (3 marks)

(b) Determine the probability that a randomly selected ingot cast by the machine has a mass less than 3.8 kg. (1 mark)

(c) Determine the mean and standard deviation of the masses of ingots cast by the machine. (4 marks)

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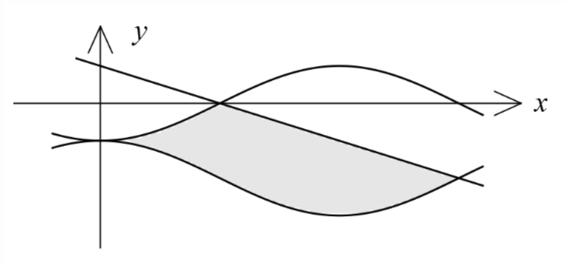
Question 14 **(7 marks)**

Functions f, g and h are defined by

$$f(x) = 10\cos\left(\frac{\pi x}{5}\right) - 20$$
$$g(x) = -10\cos\left(\frac{\pi x}{5}\right)$$
$$h(x) = 10 - 4x.$$

The graphs of these functions are shown to the right.



(a) Determine the area between $y = f(x)$, the x-axis, $x = 3.75$ and $x = 5$. (3 marks)

(b) Determine the area of the shaded region enclosed by the three functions. (4 marks)

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Question 15 **(9 marks)**

In a random sample of 125 adult male Australians, 35 were born overseas. This data is to be used to construct a 90% confidence interval for the proportion of adult male Australians born overseas.

(a) Determine the margin of error for the 90% confidence interval. (3 marks)

(b) State the 90% confidence interval. (1 mark)

(c) If 5 similar samples are taken and each used to construct a 90% confidence interval, determine the probability that at least 4 of the intervals will contain the true proportion of adult male Australians who were born overseas. (2 marks)

(d) The 90% confidence interval for the proportion of adult female Australians born overseas constructed from another random sample was $(0.202, 0.298)$. Determine the number of adult females who were born overseas in this sample. (3 marks)

Question 16 **(9 marks)**

The number of points awarded each time an online game is played is the random variable X, where $E(X) = 3.9$ and X has the following probability distribution.

x	0	1	3	6	c
$P(X = x)$	k	0.25	0.35	0.25	0.10

(a) Determine the value of the constant c and the value of the constant k. (3 marks)

(b) Calculate the variance of Y, where $Y = 10X + 5$. (3 marks)

When playing a set of 8 games, the points awarded in each game is independent of other games and a player wins a prize if the total number of points scored in the set is at least 35.

(c) A player has completed 6 games in a set and has been awarded a total of 23 points. Determine the probability that they win a prize on completion of the set. (3 marks)

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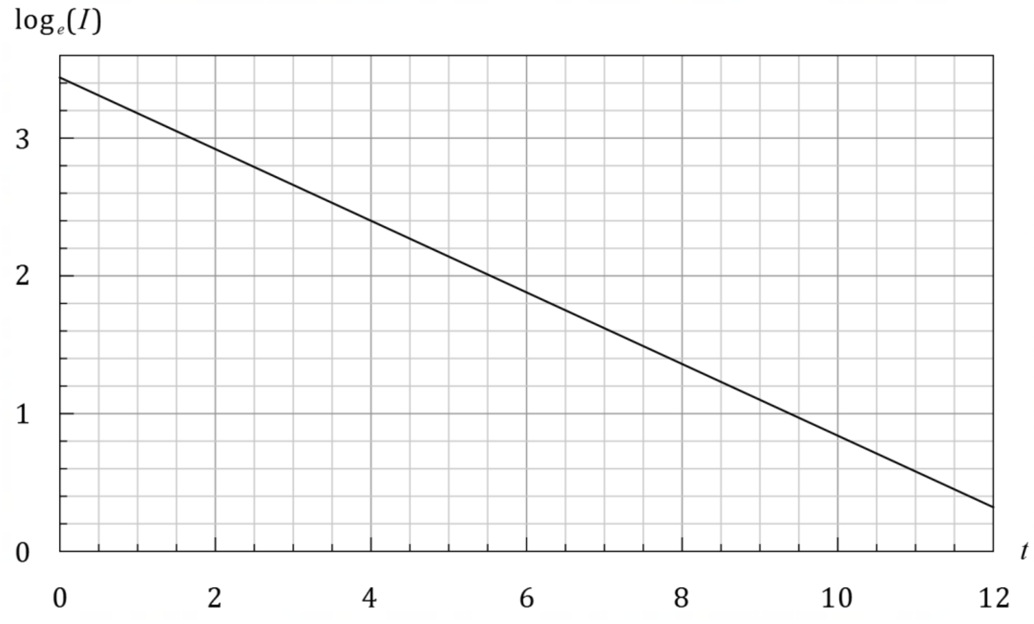
Question 17 **(11 marks)**

The turbidity index I (a measure of purity) of water being treated in tank A can be modelled by the relationship $I = 20e^{-0.25t}$, where t is the time in hours since treatment began.

(a) Express this relationship in the form $t = p\log_e(kI)$, where p and k are constants. (2 marks)

(b) Determine the time taken, to the nearest minute, for the turbidity index of the water in tank A to halve. (2 marks)

Readings of water being treated in tank B were used to construct the graph below, where a linear relationship between $\log_e(I)$ and time t exists. The line passes through the points $(4, 2.4)$ and $(9, 1.1)$.



(c) Determine the turbidity index of the water in tank B when $t = 4$. (1 mark)

(d) Determine the equation of the linear relationship shown in the graph in the form $\log_e(I) = at + b$, where a and b are constants and hence express the turbidity index I as a function of time t for the water being treated in tank B. (3 marks)

Treatment began at $1:00$ pm in tank B, and at $2:30$ pm in tank A.

(e) Determine the time at which the turbidity indices of the water in the tanks first become the same. (3 marks)

Question 18 **(8 marks)**

A small body moves along the x-axis with acceleration t seconds after leaving the origin given by $a(t) = 4.32 + kt$ cm/s^2, where k is a constant. The initial velocity of the body is -5 cm/s, and its change in displacement during the fourth second is 7.9 cm.

(a) Determine the maximum velocity of the body. (6 marks)

(b) Determine, to the nearest centimetre, the distance travelled by the body between $t = 0$ and the instant it reaches its maximum velocity. (2 marks)

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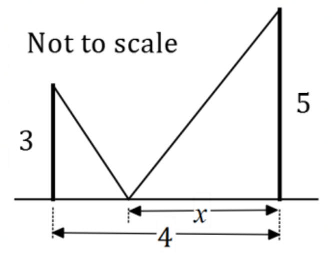
Question 19 **(7 marks)**

Two thin vertical posts, one 5 m and the other 3 m tall, stand 4 m apart on horizontal ground. A small stake is positioned directly between the bases of the posts at a distance of x m from the base of the taller post.

A length of thin wire runs in a straight line from the top of one post, to the stake, and then to the top of the other post.



(a) Calculate the length of the wire when the stake is positioned midway between the bases. (1 mark)

(b) Use a calculus method to determine where the stake should be positioned to minimise the length of wire, state what this minimum length is and justify that the length is a minimum. (6 marks)

End of questions

Supplementary page

Question number: _________

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Supplementary page

Question number: _________